

The Algebra Project: Organizing in the Spirit of Ella

ROBERT P. MOSES

MIEKO KAMII

SUSAN MCALLISTER SWAP Wheelock College, Boston

**JEFFREY HOWARD Efficacy Institute, Lexington,
Massachusetts**

This article analyzes the unique impact of civil rights

organizing — in the spirit of Ella Baker — on the grassroots effort of a community activist parent, Robert Moses. Moses, who is also a mathematician, argues that all children should have access to the college preparatory mathematics curriculum of the high schools, and that children without access to such programs are barred from acquiring the knowledge and skills necessary for participation in an economy driven by rapid technological change. In this article, the authors describe the interaction among parents, students, and teachers engaged in The Algebra Project, a seven-year ongoing effort to establish a pedagogy of mathematics that expects, encourages, and supports every student to study algebra at the middle-school level.

The United States is beginning to address, in a fundamental way, the teaching of mathematics in its middle schools. The National Science Foundation (1989), for instance, has issued a request for proposals to develop materials for middle-school mathematics instruction, that sets out the technical elements of the problem in great detail. At the heart of math-science education issues, however, is a basic political question: If the current technological revolution demands new standards of mathematics and science literacy, will all citizens be given equal access to the new skills, or will some be left behind, denied participation in the unfolding

economic and political era? Those who are concerned about the life chances for historically oppressed people in the United States must not allow math-science education to be addressed as if it were purely a matter of technical instruction.

The Algebra Project, a math-science program in Cambridge, Massachusetts, has organized local communities to help make algebra available to all seventh- and eighth-grade students, regardless of their prior level of skill development or academic achievement. The project's philosophy is that access to algebra will enable students to participate in advanced high school math and science courses, which

**Harvard Educational Review Vol. 59 No. 4 November 1989 Copyright © by
President and Fellows of Harvard College 0017-8055/89/1100-0423\$01.25/0**

423

Harvard Educational Review

in turn are a gateway for college entrance. The project offers a new curriculum and a five-step curricular process for sixth graders, which provides the following: a

smooth transition from the concepts of arithmetic to those of algebra, increasing the likelihood of mastery of seventh- and eighth-grade algebra; a home, community, and school culture involving teachers, parents, community volunteers, and school administrators in activities that support students' academic achievement; and a model of intellectual development that is based on motivation rather than ability.

The belief that ability is the essential ingredient driving intellectual development and necessary for mastering advanced school mathematics is the basis for the differentiation in mathematics curricula at the eighth-grade level as well as the widespread practice of offering eighth-grade algebra only to students who are "mathematically inclined" or "gifted." The developers of The Algebra Project have called upon the traditions of the Civil Rights Movement to assist communities in organizing a challenge to the ability model and its institutional expressions.

Traditions of the Civil Rights Movement in Mississippi

Through the Public Broadcasting System's (PBS) "Eyes on the Prize" series, the American public has been given an opportunity to revisit the Civil Rights Movement's "community mobilization tradition." Masses of people were mobilized to participate in large-scale events such

as the Birmingham campaign, the March on Washington, and the Selma-to-Montgomery March, which were aimed at achieving equal access for Southern Blacks to public facilities and institutions. The tradition is epitomized by Dr. Martin Luther King, Jr., who lifted the Movement by inspiring immense crowds in vast public spaces.

Within the Civil Rights Movement was an older, yet less well known, "community organizing tradition." This tradition laid the foundation for Mississippi Freedom Summer (1964), which revolutionized race relations in Mississippi, and the Voting Rights Act of 1965, which altered politics throughout the South during the last quarter of this century. Its leader was Ella Baker, a community organizer and fundi whose wisdom and counsel guided the Black veterans of the first wave of student sit-ins through the founding and establishment of the Student Nonviolent Coordinating Committee (SNCC).¹ She inspired in S N C C field secretaries a spiritual belief in human dignity, a faith in the capacity of Blacks to produce leaders from the ranks of their people, and a perseverance when confronting overwhelming obstacles. Baker symbolizes the tradition in the Civil Rights Movement of quiet places and the organizers who liked to work them.² Just as her spirit, consciousness, and teaching infused the Mississippi Movement, they permeated The Algebra Project from its inception.

. Fundi is a Swahili term for a person who has an expertise valued by society, and who passes on his or her art to the young by example and instruction. Ella Baker was a fundi to the SNCC workers learning the art of community organizing. 2 One such quiet place was Amite County, in a remote corner of Southwest Mississippi, where E. W. Steptoe's family welcomed Bob Moses, SNCC's first field secretary, into the community in the summer of 1961. Mr. Steptoe was president of Amite County's NAACP chapter in the late 1950s when the county sheriff raided a chapter meeting and confiscated the group's books, thus exposing the members to economic reprisals and physical danger. By the time the first wave of SNCC organizers spread out across the rural South, activities at places like the Steptoe farmhouses had ground to a halt.

424

The Algebra Project MOSES, KAMII, SWAP, AND HOWARD

Three aspects of the Mississippi organizing tradition underlie The Algebra Project: the centrality of families to the work of organizing; the empowerment of grassroots people and their recruitment for leadership; and the principle of "cast ing down your bucket where you are," or organizing in the context in which one lives and works, and working the issues found in that context.³

Families and Organizing Of central importance to the Mississippi Movement was the capacity of Black families to adopt, nurture, love, and protect Civil Rights organizers as if they were family members. This practice, known in the literature as "informal absorption," allowed SNCC and CORE (Congress of Racial Equality) field secretaries and organizers to move from place to place in Mississippi with scarcely a dollar in their pockets, knowing full well that a family welcome awaited them at the end of their journeys. The absorption of Civil Rights organizers into Black families was spiri tual gold for the Mississippi Movement, and it empowered Movement organizers with the one credential that they could never earn: being one of the community's children. This credential contradicted the label of "outside agitator," used in Mississippi by the White power structure to negate the impact of the Movement. By the same token, Movement organizers empowered their adoptive families by reinforcing and enlarging the connections between them and the larger Movement family, with its extensive networks across the land.

Grassroots People and Grassroots Leadership The Mississippi Movement's message of empowerment for grassroots people was delivered to the entire country on national television at the 1964 Democratic National

Convention by the Black sharecroppers, domestic workers, and farmers who formed the rank and file of the Mississippi Freedom Democratic Party (MFDP).

Thereafter the message of empowerment was carried by Black and White community organizers into many areas of community activity, including education, health, welfare, religion, and politics. However, neither the MFDP nor other grassroots organizations took root and flourished into a strong national movement for empowering Black people. The echoes heard from the Democratic Party to the federal government and from the religious sector to public school systems were the same: institutionalizing empowerment in the hands of Black "folk" is too frisky a notion to attract lasting political support.

The issue of community empowerment in the public schools, first raised by Black community organizers in Harlem in 1965, also found expression in White, liberal America. For example, in 1969 the Open Program of the Martin Luther King, Jr., School was established as a magnet program in the Cambridge Public Schools, in part because of the clamoring of Cambridge parents for more Open Education programs for their children, and in part because of the response to desegregation of the Cambridge schools.⁴

³ "Cast down your bucket where you are" was used by Booker T.

Washington in an address at the Atlanta Exposition, September 18, 1895.

. The King School is a large, modern facility, built on the site of a school that had served Cambridge's Black community for many years. By the late 1970s the King School housed four programs for grades K-8: a regular program composed of personnel from the former school; a magnet Open Program; and smaller bilingual and special needs programs.

425

Harvard Educational Review

"Cast down your bucket where you are" To master the art of organizing that strives to empower grassroots people, one needs to learn to "cast down your bucket where you are." In 1976 Bob and Janet Moses, both former organizers for SNCC in Mississippi, cast down their bucket in Cambridge and looked to the Open Program of the King School as a place to educate their children.⁵ What would later become the Algebra Project began in 1982 when their eldest daughter, Maisha, entered the Open Program's eighth grade.

The Algebra Project

Before 1982, Moses, whose background included teaching secondary school mathematics in New York City and in Tanzania, had been teaching math to his children at home. Maisha, now a Junior at Harvard University, recalls these lessons, conducted weekly during the school year and daily during the summer and vacations:

Doing math at home was always a lot harder than math at school. It was somewhat like a chore. In our family, extra reading with my mom when we were much younger and math with my dad was part of our responsibility in the family, like taking out the garbage or doing the laundry.

Moses faced a familiar challenge: the resistance of adolescent children to performing what they regarded as a "household chore." Maisha explains:

As we were getting older, it was a lot harder to get us to do math at home. We battled a lot more and complained. "Why do we have to do this? No one else has to do this." Dad would say, "It's important. I want you to do it. You need to do it." But we wouldn't be satisfied. I didn't really want to do it. Dad would have to sit there and force answers out of me. Finally he decided that the only way to get me to do algebra was to go into school.

In the Fall of 1982, Mary Lou Mehrling, Maisha's

eighth-grade teacher, invited Moses into her seventh/eighth grade classroom to work with Maisha and three other eighth graders on algebra. That spring Maisha and two others took the Cambridge citywide algebra test, offered to students who wish to bypass Algebra I and go directly into Honors Algebra or Honors Geometry in ninth grade. All three passed, becoming the first students in the history of the King School to be eligible to pursue the honors math and science curriculum at Cambridge's only high school, Cambridge Rindge and Latin.

With one eye on his eldest son, who was about to enter the Open Program's seventh grade, Moses decided to continue working the next year (1983-84) with Mehrling and another seventh/eighth grade teacher. The number of eighth graders studying algebra with Moses was increased to nine. Partway through the year, the teachers selected seven seventh graders whom they thought likely to be-

Because some but not all authors of this article are also the subjects of discussion, we have chosen

to use third-person references throughout to avoid confusion. The fourth student opted to go to a private high school, and did not take the test.

The Algebra Project MOSES, KAMII, SWAP, AND HOWARD

gin algebra the following year, creating the first group of "high ability" seventh graders for Moses to direct. In the spring, all nine of Moses's eighth graders took the citywide algebra test, and six passed.

In the following year the program expanded again, but it was no longer quite the same. As early as 1983-84, it was evident that in spite of the commitment to meeting the educational needs of all its pupils, mathematics instruction in the Open Program was unwittingly skewed along racial lines.⁷ Children in the two seventh/eighth grade classrooms were clustered into separate "ability" groups: above-grade-level tracks primarily composed of middle-class Whites; below- grade-level tracks made up almost exclusively of Blacks and other children of color; and grade-level tracks that were racially mixed. The Open Program's system of ability groups effectively shunted most students of color onto the non- algebra track, imbuing too many youngsters with the self-fulfilling notion that little was expected of them.

Additionally, Moses and Mehrling became aware that some high-achieving Black males felt uncomfortable joining the algebra group, for it meant being separated from their friends, who were on other math tracks. On the whole, young people feel the need to be as similar to their peers as possible. Separating academically talented adolescents from their peers for the sake of participation in the academic "fast track" potentially aggravates the anxiety that accompanies adolescents' identity development. Moreover, enduring attitudes toward math are shaped by math instruction at the seventh- and eighth-grade levels. Traditionally, very few new math principles are introduced in these two grades, when attention focuses instead on review (Usiskin, 1987). Moses and Mehrling hypothesized that using the seventh and eighth grades to lay a groundwork of competence in algebra might enhance students' general self-confidence, and provide them with the mathematical background necessary for advanced high school courses.

The Mississippi Movement's organizing tradition utilized everyday issues of ordinary people and framed them for the maximum benefit of the community. In Mississippi the issue was the right to vote; technically: "What are the legal, judicial, political, and constitutional obstacles to the right to vote? How can we initiate court cases, introduce legislation, and mobilize political support to remove these obstacles?" SNCC and CORE workers

pursued this by establishing beachheads, through Black families, in the most resistant counties throughout the state. But the Mississippi organizers did something of even greater importance, and that was to conceive of the issue of voting in its broadest political sense. Midway through voter registration efforts, they began to ask themselves and the Black community: "What is the vote for? Why do we want it in the first place? What must we do right now to ensure that when we have the vote, it will work for us to benefit our communities?" After the organizers and key community groups had worked and reworked these and other questions, they shifted the organizing strategy from increasing voter registration to laying the basis for a community-based political party, which eventually became the Mississippi Freedom Democratic Party. Creating

See Delpit (1986) for a discussion of the differences between the instructional needs of mainstream and minority children. See Fordham (1988) for a discussion of the tensions high-achieving Black students feel when they strive for academic success.

a new political party became the Mississippi Movement's focus, because of its greater potential for involving community people in a substantive long-term effort. Participants would come to own the political questions and their responses to them. In the Open Program the everyday issue was teaching algebra in the seventh and eighth grades. Moses, the parent-as-organizer in the program, instinctively used the lesson he had learned in Mississippi, transforming the everyday issue into a broader political question for the Open Program community to consider: What is algebra for? Why do we want children to study it? What do we need to include in the mathematics education of every middle school student, to provide each and every one of them with access to the college preparatory mathematics curriculum in high school? Why is it important to gain such access?

By linking the content of math education to the future prospects of inner-city children, Moses transformed what had previously been a purely curricular issue into a broader political question. Drawing on his experience as an organizer, educator, and parent, Moses transformed the dialogue in the Open Program among parents,

teachers, and school administrators into one that centered on questions that would get at the heart of educational practice: How can a culture be created in the Open Program in which every child is expected to be as good as possible in his or her mathematical development? What should the content of middle school mathematics be? What curricular processes make that content available to all students?

A cornerstone of the evolving Algebra Project thus became the expectation that every child in the Open Program could achieve math literacy, an ethos powerful enough to suffuse both the peer and adult culture. The components of this effort included changing the content and methods of teaching math, involving parents in activities that would enable them to better support their children's learning, teaching students to set goals and motivating them to achieve, and reaching out to Black college graduates in the Boston area who would serve as tutors and role models of academic success.

Teachers as Learners From the beginning, Mehrling and Moses modeled the notion that there is no shame in confessing ignorance — if it is the first step in learning. Mehrling, an ex-music teacher, took courses in mathematics, beginning with algebra, and eventually achieved state certification in math. But she did something more profound: she turned her inexperience

with math content into a component of learning by adopting a position of mutual inquiry with her students, and by presenting herself to them as a learner. As she states, she "developed methods of responding to students' questions that helped both the students and me to think through the problem." As she had questions, she would ask Moses for help, on the spot.

Presenting myself as a learner, in front of my students, helped me to understand what they were experiencing, and helped them to feel comfortable asking for help. Students no longer felt threatened if they did not understand a problem or a concept, for they saw that we all were learners and we all learn in different ways.

It was only in Mississippi, where the entire state was structured along a community organizing tradition, that the issue of the right to vote was perceived as a broad political question.

428

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

Because Mehrling presented herself openly and honestly as

a student of the subject she was teaching, she was able to help build her students' confidence. She overtly transmitted the message, "if I [your teacher] can risk embarrassment to learn this subject, surely you can, too." But she also conveyed to them a powerful latent message:

I am confident that people who don't know this subject can learn it; to learn it they have, at all times, to be ready not to pretend to understand what they do not truly understand; to learn it they must be comfortable asking for help and willing to risk embarrassment.

Mehrling's message recapitulated a memorable message that Fannie Lou Hamer and others conveyed at the height of the MFDP challenge to the Democratic National Convention of 1964 — confidence that people who did not know the business of politics could learn it by asking direct questions and risking embarrassment. Each confronted their inexperience with honesty and integrity, turning potential liabilities into strengths.

Involving Parents

From its inception, the Open Program had evolved a set of policies and practices that encouraged parents' active involvement in staff hiring, curriculum development, observation and evaluation of teachers, and governance and administration of the school. Parental involvement in the Algebra Project grew naturally in this context.

Parents who served on the Program's seventh/eighth-grade committee in 1984-85 concluded that decisions about studying algebra in the seventh and eighth grades could not be left up to individual sixth graders to make. These children were too young to fully understand the long-range implications of their decisions for college entrance. Nor should such decisions rest solely with the teachers, curriculum coordinators, or school or districtwide administrators, each of whom had their own ideas about who should study algebra and in which grade. Rather, parents needed to be involved in making educational choices for their children at both individual and policy-making levels. They also had to be better informed about details of the middle school math curriculum, so that they would be able to make informed decisions and to protect the best interests of their children.

During the Spring of 1985, a parent from the Open Program's seventh/eighth-grade committee collaborated with Moses to distribute a letter to the parents of all the sixth graders, asking whether they thought that every seventh grader should study algebra, and whether they thought their own child should study algebra in the seventh grade. In reply, a few parents thought that some seventh graders probably weren't ready, but no parent thought his or her own child should be denied access to algebra in the seventh grade. Exposing the contradictions between parental assessments of their children's capabilities and curricular assumptions at the community level provided a means for building consensus around educational outcomes for all children.

This was the catalyst for inviting all Open Program children entering the seventh grade in the Fall of 1985 to study algebra three times a week. With the exception of a few eighth graders who in their teacher's judgment "were not ready," the invitation to study algebra was extended to the entire eighth grade as well. The

429

Harvard Educational Review

consensus statement from parents launched a change in school policy and culture. Currently, every Open Program student is expected to study algebra in the seventh and eighth grades.

As the Project evolved, parental participation increased as parents volunteered in classrooms and participated in workshops on student self-esteem and achievement. Parents from throughout the King School were invited to attend "Honors Bound" parent groups, which prepared students of color to accept the challenge of taking honors courses in high school, and created a home-school culture that would nurture and support serious intellectual effort. A Saturday morning algebra course for parents was offered, teaching algebra in the same way that it was being taught to

their children.

Parents who took algebra during the Saturday classes committed themselves to making the Project "theirs" in a fundamental sense. A grateful parent captured the multiple dimensions of this experience in a 1987 letter to the Cambridge School Committee:

. . . this program exemplifies to me all that I hope most for in the education of my daughter and other young people in our community: a positive orientation to learning; a rich understanding of advanced mathematics; recognition of the relationship between what is learned in the classroom and what goes on in life; and a sense of personal empowerment.

As a sixth grader in her first year in the program, my daughter began to overcome her fear of math and distorted perceptions of what she is capable of doing and why it is important. I believe this was due to several factors including the climate of learning in the classroom (in part, a sense that students, teachers and aides alike were learning together); the demystification of the subject by relating it to life experiences; and by the fact that her mother, along with other parents and community members, was simultaneously overcoming latent math panic by taking the course on Saturdays.

This experience not only helped me understand the program (and learn math); it also greatly enhanced my comprehension of the life of the school and neighborhood community and of problems that as a citizen I can help to resolve . . .

Parents were barraged with letters and opportunities to talk, to ask questions, and to join in planning, all as an acknowledgement of the centrality of parents in the construction of a home-school culture of high achievement.

Creating a New Teaching and Learning Environment for Math

As an adjunct to opening up algebra to all seventh and eighth graders in 1985, ability grouping was replaced with individual and small group instruction. Students were taught skills for learning hard material "on their own." In conferences with teachers, students were asked to set their own short-term objectives (for example, deciding how many lesson sets they wished to complete each week), and longer-range goals (for example, deciding to prepare for the citywide test). Parents were informed about the goals, and were asked to sign their child's goal statement each semester. The pace and scope of students' mathematical studies therefore came under student control. Mehrling tells a story that reflects the individual and group motivation that such goal-setting can foster:

430

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

Andrea spoke up at one of our first meetings and said, "I'm going to do four lessons a week because I want to finish such-and-such by the end of seventh grade, so I can finish the book by the end of the eighth grade, so I can be in honors geometry in the ninth grade." This was a twelve-year-old. The others looked at her — this hadn't come from a teacher — and said, "Are you crazy?" She said, "That's what I'm going to do." Bob [Moses] was there, and he started to frame for them why what Andrea had just done was a very mature and farsighted act, and how maybe they weren't ready to do that yet. But it gave Andrea a lot of support and affirmation for having said that in the group. And it changed what the others were going to say next. Everything from then on was in terms of Andrea: "Well, I'm not going to do quite what Andrea is, but . . ."

Students also learned to work harder than they had before. They were encouraged to develop habits of concentration, patience, and perseverance in approaching their daily math work. Students decided which of several resources to consult — the textbook, the instructor, or a peer — when they had a question or ran into difficulties in solving a problem. Teachers met with small groups for brief lessons on specific concepts, and regularly held small-group review sessions. Reflecting on this decision, Mehrling recently explained:

Adolescent learners can sometimes interrelate with materials, and it's not nearly as threatening as interacting with an adult. If they can go to an adult to ask a question about the materials, when they're ready to go to an adult, it's wholly different from being in a group, being pinpointed and put on the spot, and feeling vulnerable about the pieces they don't have in place yet. Once they start to interact with materials, they get not only very possessive of them, but very reluctant to go back to any kind of teacher-directed lessons. They're empowered, in a curious way, around materials — something I would never have even thought about. The Open Program generally is a very teacher-intensive kind of program. We motivate, we bring in materials from everywhere, and our teaching is interpersonal. We discovered at the seventh and eighth grade level that that was one of the problems with students who felt vulnerable: It put them on the spot.

As part of the new curricular, pedagogical, and social environment for studying math, the seventh- and eighth-grade teachers assumed the role of "coach" as opposed to "lecturer" in their relationship with students.

The Project produced its first full graduating class in the Spring of 1986. When they entered high school the following autumn, 39 percent of the graduates were placed in Honors Geometry or Honors Algebra. Not a single student in that cohort ended up at Cambridge Rindge and Latin School in lower-level math courses, such as Algebra I.

Curricular Expansion

By 1986 attention turned to the preparation of students for seventh-grade algebra. With all students in the seventh and eighth grades taking algebra, lower grade teachers began to question the adequacy of their own math curricula as preparation for algebra. To address this question systematically, the entire staff of the Open Program participated in a year-long institute centered on the question of math literacy.

431

Harvard Educational Review

After the institute, teachers at all levels (K-8) implemented new curricula in mathematics, appropriate for the age and grade levels that they taught. Some teachers found it unsettling to devise their own curricular practices around the needs of children and their own teaching styles. The results of the Algebra Project suggest that flexibility leads to better pedagogy. For example, when fifth/sixth grade teachers tried

a materials-centered approach with sixth graders that had worked very well at the seventh/eighth grade level, they found that younger children, accustomed to more teacher-centered instruction, needed more teacher-child and small group interaction in the sixth-grade transition curriculum. The teachers modified their classroom technique, but retained the principle of encouraging greater self-reliance in finding answers to problems. Improved adaptation of curriculum was itself beneficial. But equally important, this process gave teachers the same sense of empowerment experienced by students. Teachers who participated in the innovation and trained themselves in how to present the curriculum were more likely to understand, appreciate, and foster the skill of self-education that was central to the Algebra Project. One teacher explained:

Bob was affirming what we were doing while he was helping us change. He didn't come in and say, "We're throwing this out, it's junk." He came in and said, "You guys are great. Wanna try something different?" When we asked, "How will it work?" he turned it around and asked, "Well, how do you think it should work? What do you want to have happen?" He didn't really give us a way, which admittedly was frustrating, but it also gave us ownership around it. Bob didn't have all of the answers. At first I was really annoyed that he was making me go through this process. I kept saying, "Bob has an agenda. Why doesn't he tell us? We're wasting so much time!" But he knew that it had to come from us. He knew he couldn't impose, because he didn't know what would work.

He wasn't a classroom teacher. He just had the vision. If he could help us catch the vision, we would make it work.

A second outcome was that Moses agreed to develop a curriculum for the sixth grade that would provide a conceptual transition from arithmetic to algebra. The main features of what has come to be called the Algebra Project, and the philosophy that guided its construction, are discussed below.

What to Teach and How to Teach It

The opening of algebra to everyone in 1985-86 gave Moses the opportunity to work closely with several students who had great difficulty with the initial chapters of the algebra textbook. In particular, one Black male student took many months to complete the first few lessons. Moses wondered precisely where the student's conceptual knot lay. Was it possible to lead the student from arithmetic to algebra by mapping a conceptual trail, beginning with concepts that were obvious, and proceeding by equally obvious steps?

After working with a number of students who were having difficulty, Moses came to the conclusion that the heart of the problem lay in their concept of number. In arithmetic, the distinctive feature of a number is magnitude or quantity. In algebra a number has two distinctive features: one is quantitative; the other is qualitative and must be explicitly taught. Students of arithmetic have in their minds one question that they associate with counting numbers: "How

many?" Stu-

432

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

dents of algebra need to have two: "How many?" and another question, such as "Which way?" as points of reference for the intuitive concept of opposites. Children understand the question, "Which way?" from their early years, but it is not a question that they associate with number. The number concept used in arithmetic must be generalized in algebra, and failure to make this generalization blocks students' understanding. Once students have generalized their concept of number, they must also generalize their knowledge of basic operations such as subtraction.

Moses gradually arrived at a five-step teaching and learning process that takes students from physical events to a symbolic representation of those events, thereby accelerating sixth graders' grasp of key concepts needed in the study of algebra.¹⁰ The five steps are:

1. Physical event

- 2. Picture or model of this event**
- 3. Intuitive (idiomatic) language description of this event**
- 4. A description of this event in regimented English**
- 5. Symbolic representation of the event**

The purpose of the five steps is to avert student frustration in "the game of signs," or the misapprehension that mathematics is the manipulation of a collection of mysterious symbols and signs. Chad, a young Black seventh grader, recently looked up from reading a page in the first chapter of a traditional algebra text and said to his mother, "It's all just words." For too many youngsters, mathematics is a game of signs they cannot play. They must be helped to understand what those signs really mean, and construct for themselves a basis of evidence for mathematics. When middle-school students use the five-step process to construct symbolic representations of physical events (representations that they themselves make up), they forge, through direct experience, their own platform of mathematical truths. Their personally constructed symbolic representations enter into a system of mathematical truths that has content and meaning.

At the Open Program, students initiate this process with a trip on the Red Line of Boston's subway system (the physical event). This experience provides the context in which a number of obvious questions may be asked: At what station do we start? Where are we going? How many stops will it

take to get there? In what direction do we go? These questions have obvious answers, forming the basis for the mathematics of trips. When they return, students are asked to write about their trip, draw a mural or construct a three-dimensional model, make graphs for trips that they create, and collect statistical data about them. The purpose is to

¹⁰ This model is a synthesis of ideas derived from three sources. The first was the Open Program itself. Moses observed teaching practices in the Open Program and attended workshops with teachers in which Virginia Chalmers and others explained the teaching and learning ideas that they had developed for primary grades. The second was Quine's (1981) notion of "mathematization in situ." "A progressive sharpening and regimenting of ordinary idioms: this is what led to arithmetic, symbolic logic, and set theory, and this is mathematization" (p. 150). Quine insisted that "set theory, arithmetic, and symbolic logic are all of them products of the straightforward mathematization of ordinary interpreted discourse . . ." (p. 151). The third source was Dubinsky (1987), who shared his insight that in the future, mathematics education would center on a "fixed curricular process" rather than a "fixed curriculum."

fuse in their minds the two questions "How many?" and "Which way?" and to anchor these questions to physical events.

Students then use this process to explore the concept of equivalence, in the broad cultural context of everyday events such as cooking, coaching, teaching, painting, and repairing. They explore any concept in which an object A is substituted for another object B to achieve a certain goal. They conclude the discussion of equivalence in subway travel with open-ended constructions of equivalent trips, leading to an introduction of displacements as "trips that have the same number of stops and go in the same direction."

Once displacements are introduced, they investigate the concept of "comparing" as a prelude to generalizing their concept of subtraction. Most algebra texts introduce subtraction as a transformed addition problem. Students are asked to think of subtraction ($3 - (-2) = + 5$) as "adding the opposite" or "finding the missing addend" ($3 - ? = 5$), which provides one group of signs as a reference for another. But students look for concrete experiences, pictures, or at least a concept, to link directly to algebraic subtraction. The problem is compounded because students have over-learned "take-away" as the concept underlying subtraction. In algebra, "take-away" no longer has a

straightforward application to subtraction. Within a couple of months of beginning algebra, students confront subtraction statements which have no discernable content, have only indirect meaning in relation to an associated addition problem, and are not at all obvious.

To give additional content, meaning, and clarity to subtraction in beginning algebra, students begin with the physical event of comparing the heights of two students, Coastcoast who is six feet tall, and Watchme who is four feet tall. The class works with a picture of this event, generating questions that can be used to compare heights:

1. Which one is taller?
2. What is the difference in their heights?
3. How much shorter is Watchme than Coastcoast?
4. Who is shorter?
5. How much taller is Coastcoast than Watchme?

In arithmetic there are two subtraction concepts, the concept of "take-away" and the concept of "the difference between." The latter provides the appropriate entry into subtraction in algebra, as illustrated in the above set of questions. Students will readily identify an answer to the second question by subtracting to find the difference in the heights. This prepares them to accept subtraction as the best approach to answering comparative questions — questions that belong to algebra and not arithmetic.

The answers to these questions are carefully processed in three stages: intuitive language, regimented English, and symbolic representations. "How much taller is Coastcoast than Watchme?" is explored in the following way:

- Intuitive language: "Coastcoast is two feet taller than Watchme."
- Regimented English: "The height of Coastcoast compared to the height of Watchme is two feet taller."

434

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

Symbolic representations: (5a) $H(C)$ compared to $H(W)$ is 2'

$$(5b) H(C) - H(W) = 2'$$

$$(5c) 6' - 4' = 2'$$

(that is, 6' is 2' taller than 4')

"How much shorter is Watchme than Coastcoast?" proceeds along a similar track.

**Intuitive language: "Watchme is two feet shorter than Coastcoast." Regimented English: "The height of Watchme compared to the height of Coastcoast is two feet shorter."
Symbolic representations: (3a) $H(W)$ compared to $H(C)$ is 2'**

(3b) $H(W) - H(C) = 2'$

(3c) $4' - 6' = 2'$

(that is, a height of 4' is 2' shorter than a height of 6')

This way of comparing physical quantities is easily reinforced with work stations at which students compare weights, lengths, temperatures, and speeds. They may return to their experience on the subway to compare positions of stations on the Red Line, using the following model.

When asked, "What is the position of Harvard compared to Kendall?" students work through the following steps:

Intuitive language: "Harvard is two stops outbound from Kendall." Regimented English: "The position of Harvard compared to the position of Kendall is two stops outbound."

Symbolic representations:

(a) $P(H)$ compared to $P(K)$ is 2

(b) $P(H) - P(K) = 2$

In a similar way the question, "What is the position of Kendall relative to Harvard?" yields

$P(K) - P(H) = 2$

435

Harvard Educational Review

As soon as integers are introduced as a system of coordinates, students are ready to generate their own subtraction problems. The notion of an arbitrary point of reference having been introduced earlier, systems of coordinates are assigned to the stations, with the zero point alternately assigned to various stations. Each assignment

generates a different subtraction problem for the question, "what is the position of Harvard relative to Kendall?"

(a) $P(H)$ compared to $P(K)$ is 2

(b) $P(H) - P(K) = 2$

(c) $-3 - (-1) = -2$

By similar reasoning, the question, "what is the position of Kendall relative to Harvard?" yields

$P(K) - P(H) = 2$

$-1 - (-3) = +2$

The opposite comparisons [$P(H)$ compared to $P(K)$, and $P(K)$ compared to $P(H)$] lead to opposite expressions [$(-3) - (-1)$,

and $(-1) - (-3)$] as well as opposite integers $[(-2) \text{ and } (+2)]$, in a way that gives direct, intuitive meaning to subtraction of integers and provides students and teachers alike with control over the generation of simple subtraction problems and equations. The curriculum and curricular process used in the sixth grade have made algebra accessible for all middle school students. The Project has demonstrated that all seventh- and eighth- grade students in the King School's Open Program can study algebra, and that the entire school community expects them to do so.

Community Participation in Creating a Culture of Achievement

For youngsters who have felt excluded from the culture of academic achievement in school, the expectation that they, too, can learn is crucial. During the 1987-88 school year, the Project's response to children who did not think they were likely to succeed in math was to institute a series of measures designed to create a culture of mathematical and scientific literacy, not only in the Open Program, but in other programs within the King School as well. The Seymour Institute for Advanced Christian Studies, a service organization conceived by Black Harvard graduates to support community-based development in urban areas, provided Black role models to go into classes to tutor students and to run before-school algebra study

436

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

halls four mornings a week. The study halls were open to seventh and eighth graders from all of the King School's four programs. The tutors, who came from Harvard, MIT, Wentworth Institute, and Boston University, established relationships with individual children and became role models of academically successful young adults for seventh and eighth graders to emulate. A Harvard Law School student and tutor wrote:

I have been impressed by the fact that these seventh and eighth graders are able to read and understand their math textbooks, already have some understanding of algebraic concepts, and are willing to come out at 7:30 a.m. in order to work on their mathematical skills. . . . The students in the Algebra Project are able to help themselves, and each other, by using their books. Helping each other has another important role in the Project. I believe that it is their friendships that keep them coming to early morning study halls; relationships that support educational achievement are being established outside the classroom.

As The Algebra Project developed, the message that each child could learn was more systematically articulated by the Efficacy Institute.¹¹ Emphasizing confidence and effective effort as key ingredients in the process of intellectual development, the Efficacy model provides educators, parents, and students with an explicit alternative to the ability model of learning. Efficacy assumes that children, who are well enough endowed to master the fundamentals of language at an early age, are fully capable of learning mathematics. In order to learn, children are required to marshal effective effort. They must learn to work with commitment, focused attention, and reliable strategies. When learning is perceived as a function of effective effort, one seeks factors inhibiting children when they are having difficulties learning or understanding a concept, rather than "disabilities" that disallow learning.

Many children of color learn from an early age that there are doubts concerning their capacity to develop intellectually. Messages communicated from school (low ability placements in the primary grades), from peers (pervasive anti-intellectualism within the peer group), and the media (expectations of inferiority) all serve to impress upon them that they may not be up to the task of advanced studies. The lack of confidence engendered by the internalization of these messages shapes the meaning of any failure ("I guess this proves I'm not smart") and undermines the capacity to work ("Why bang my head against the wall if I'm unable to learn the stuff anyway?").

To redress these circumstances, Efficacy works to plant an alternative idea in the child's mind: "If I work hard enough, I can get smart":

	Development
Confidence	(Work hard)
(Think you can)	(Get smart)
Effective Effort	(Get smart)
Development	

Emphasis is placed on the process of development and some measure of control is returned to the child.

11 The Efficacy model of intellectual development is based on motivation. The role of motivation in self-development was studied by Jeffrey Howard, Director of the Efficacy Institute, who, in collaboration with educators, developed the model summarized here.

437

Harvard Educational Review

Teachers are the carriers of Efficacy ideas, and it is to them that responsibility falls for building confidence and shaping strong effort in children. Teachers attend an intensive,

five-day seminar to learn the Efficacy model of development and study its implications for their own teaching. They are then provided with formal curriculum to use with their students over the course of an academic year. The curriculum gives teachers and students a shared language and a conceptual frame work for reworking questions, such as why a particular child has been unable to "do math" in the past. The teacher is able to impress upon the child that learning is a function of effort, not of innate ability. The curriculum helps the students to raise their consciousness, so they can affirm for themselves their own need for self-development. Such affirmation on their part is a critical prerequisite to confronting obstacles to their own development and acquiring attitudes and habits that will ensure success in many endeavors, including the algebra program.

In 1988, a sixth-grade teacher in the Open Program began teaching the Efficacy curriculum to all the sixth graders twice a week. She explains:

We all consider ourselves to be good teachers, and yet we know that we are failing some students. Bob talked to us about a way that could help us to help those children achieve. We realized what that will mean not only to those students but to all of the children in our classrooms, and from there, what that will mean to the community at large.

The Project Continues

The Algebra Project continues at the Open Program. The Efficacy and Algebra curricula are taught to sixth graders, and algebra is studied by all seventh and eighth graders. The project is now challenging other schools to make the political decision to alter their own math curricula. For example, discussions are proceeding with administrators and teachers in Boston, where three schools have volunteered to experiment with both the Efficacy and Algebra Project curricula and receive training in their implementation. Moses has also begun to train selected middle school teachers in Atlanta. Currently, the Project is exploring relationships with school systems in other cities.

Conclusion

Community Organizing and Educational Innovation

The community organizing approach to educational innovation differs from traditional educational interventions in several important ways. The principle of "casting down your bucket where you are" stands in marked contrast to research programs originating in universities, where scholars design interventions they hypothesize will result in outcomes they articulate in advance and that are replicable. Researchers in universities and consulting firms must have well-designed, highly articulated interventions in order to convince funding agencies that their projects have promise. Depending upon the focus of the investigation, the researcher generally targets selected neighborhoods, schools,

or organizations for participation due to their demographic or similarly quantifiable characteristics. Additionally, researchers have intellectual roots in their own disciplines, and view

438

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

problems through lenses that are consonant with their disciplines, rather than through the eyes of a community.

In contrast to the university-based researcher, the organizer working in the tradition of Ella gradually becomes recognized by community members as having a commitment to their overall well-being. The organizer immerses him- or herself in the life of the community, learning its strengths, resources, concerns, and ways of conducting business. The organizer does not have a comprehensive, detailed plan for remedying a perceived problem, but takes an "evolutionary" view of his or her own role in the construction of the solution. He or she understands that the community's everyday concerns can be transformed into broader political questions of general import. The form they will take is not always known in advance. Once political questions are identified, the organizer's agenda must remain

simultaneously focused and fluid — sharply focused on the long-range goal, but fluid with respect to how the goal will be attained. The organizer seeks out views of community participants who have strong interests in the issue, and informally educates community members who are uninvolved but whose interests are at stake. It is the organizer's task to help community members air their opinions, question one another, and then build consensus, a process that usually takes a good deal of time to complete.

Improving the mathematics curriculum and curricular process in a middle school has gradually become the focus of the Algebra Project. At the outset Moses did not know that the Project would become a vehicle for raising questions about ability grouping, effective teaching for children of color, or the community's roles in educational decisionmaking. He did not imagine that it would trigger an interest in teaching algebra to inner-city middle school students beyond his daughter's classroom.

As we have seen, the Program's innovations relied on the involvement of the entire community: teachers, parents, school administrators, students, tutors, and consultants from the Greater Boston community. In her review of programs that have been helpful in breaking the cycle of disadvantage, Lisbeth Schorr (1988) highlights the importance of comprehensive, flexible, and intensive approaches to

reform: ¹²

Many interventions have turned out to be ineffective not because seriously disadvantaged families are beyond help, but because we have tried to attack complex, deeply rooted tangles of

troubles with isolated fragments of help, with help rendered grudgingly in one-shot forays, with help designed less to meet the needs of beneficiaries than to conform to professional or bureaucratic convenience, with help that may be useful to middle-class families but is often irrelevant to families struggling to survive. (pp. 263-264)

¹² As a participant of the Harvard University Working Group on Early Life and Adolescence, Lisbeth Schorr believed that with the knowledge currently available, society could prevent the damaging outcomes for adolescents associated with disadvantage, such as teenage pregnancy, juvenile crime, school failure, and unemployment. She visited an array of health and education programs that were successful in interrupting the cycle of disadvantage and discovered that what the programs had in common was a comprehensive, flexible, and intensive approach to reform.

439

Harvard Educational Review

The work of discovering new solutions, building a broad base of support, and overcoming barriers takes time. Moses's effort to work with teachers, parents, and administrators to transform the

middle school mathematics curriculum and curricular process in the Open Program began seven years ago. We note that it took fifteen years for James Comer's efforts at comprehensive reform in two New Haven schools to yield striking improvements in test scores (Comer, 1988; 1980).¹³ Durable reforms are possible, but there are no shortcuts in bottom-up implementation.

In the Open Program, faculty volunteered to participate, committing themselves to working together to discover better ways to teach math, and struggling to reach consensus. Parents were deeply involved as learners, supporters, contributors, and decisionmakers. Students voluntarily set goals for themselves and came to 7:30 a.m. study halls four mornings a week. School administrators supported teachers as they tried out new strategies, worked to secure funding, and acted as spokespersons for the Project. Strengths of various contributors were recognized, and they were empowered to adapt, create, and evaluate their progress in attaining a shared vision.

Others have learned that it is through struggling with a problem and shaping the solutions that commitment to change really occurs. Schorr (1988) reports:

Dr. Comer wanted to make sure I understood that the essence of his intervention is a process, not a package of materials, instructional methods, or techniques. "It is the creation of a sense of community and direction for parents, school staff, and students alike." (p. 234)

Comer is pointing to the fact that significant innovations must transform the culture, and transformation requires a broad base of voluntary support. It is crucial that participants have time to

understand an idea, explore their commitment, and adapt the innovation to their needs.

Henry Levin (1988) also emphasizes the importance of process.¹⁴ He states:

Underlying the organizational approach are two major assumptions: First, the strategy must "empower" all of the major participants and raise their sense of efficacy and of responsibility for the outcomes of the school. Second, the approach must build on the considerable strengths of the participants rather than decrying their weaknesses. (p. 5)

¹³In 1968 James Comer, a psychiatrist at the Yale Child Study Center, began a program of reform in the two New Haven schools that had the lowest achievement scores and the worst attendance and behavior records in the system. Today, although the community is still impoverished, these demonstration schools now boast top achievement scores in the New Haven system (third and fourth), no serious behavior problems, and superior attendance records. The critical components of the reform, now disseminated to fourteen other sites, include a School Planning and Management Team (composed of the principal, parents, and teachers), a mental health team that provides coordinated services to children in conflict, and extensive parent involvement. ¹⁴Henry Levin is the director of the very successful Accelerated Schools Project in the San Francisco Bay Area, whose mission is "bringing children into the educational mainstream so that they can fully benefit from future schooling and their adult opportunities" (1988, p. 3).

440

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

Many will find it useful to follow the precept "cast down your bucket where you are," as Jaime Escalante did in Los Angeles when he began offering calculus to disadvantaged youth. The starting point for reform is less important than whether the issue is powerful and inspiring enough to generate enthusiasm, reveal broader political questions, compel devoted leadership, and serve as a vehicle for community commitment.

Funding to Support Innovation

The Algebra Project would not have developed as it did had it not been for the MacArthur "no strings attached" Fellowship that allowed Moses to work in the Open Program for five years, without having to account for the way he spent his time. Subsequent funding has been difficult. For eighteen months Wheelock College in Boston supported Moses as he looked for resources to provide release time for teachers, cover materials and reproduction costs, and secure consultation from the broader academic community. Moses is still spending an

enormous amount of time trying to secure long-term funding to support the continuation and dissemination of the Algebra Project.

Finding support can be a depleting struggle for many innovative efforts. National funding sources are hesitant to fund projects with grassroots leadership, a community focus, a long time-frame, and a philosophy that casts educational issues in political as well as technical terms. Declining state and local budgets also threaten commitment to comprehensive, long-term reforms. But only when major political questions are addressed (for instance, that all children can benefit from and should have access to algebra in their middle school years) can we discover the most appropriate ways to organize knowledge, develop curriculum, and encourage home, school, and community participation.

Transforming School Culture

Teachers and parents in the Open Program came to believe that ability grouping in mathematics seriously impaired the capacity of middle school students of color and females to learn as well as they might. Questioning the policy was the first step towards comprehensive change. Others concur that differentiating students harms those who are disadvantaged or placed in lower tracks. ¹⁵After articulating a vision of high expectations in algebra for all students, participants worked to transform the culture of the school, so that policies, teaching strategies, and the Efficacy curriculum could together help students.

The project speaks to the importance of family as a link to school success. Henderson (1987) concludes her review of research

concerning parental involvement in student achievement by categorically stating that "the evidence is beyond dis-

¹⁵Levin (1988) argued that the major reason for the failure of many disadvantaged children is low teacher expectation, which in turn leads to pull-out programs based on tedious drill-and-practice curricula. Peterson (1989) conducted a study in Utah, concluding that ability grouping is harmful to remedial students, and that participation in accelerated programs is a more effective route to higher achievement.

441

Harvard Educational Review

pute: parent involvement improves student achievement" (p. 1). This finding holds for middle- as well as low-income families, at different grade levels, and in a broad spectrum of interventions. As the U.S. population becomes more diverse, it is absolutely fundamental that schools join with families to define and support school success. Continuity between home and school must be forged for all children, and we must draw on the strengths and resources that families can provide.

Curriculum and Curricular Process

Among the strengths of the faculty and volunteers in the Open Program was their curiosity about why some children were not succeeding in mathematics, and their willingness to explore the possibility that their own teaching strategies might be a factor. Moses and the teachers became classroom researchers — analyzing student errors, locating conceptual knots, and experimenting with materials and teaching processes that might improve students' mathematical development. A sixth-grade transition curriculum that allows students to relate everyday experiences to mathematical concepts represented symbolically should be disseminated widely.

In 1964 national attention was focused on the disenfranchised citizens of the South. In 1989 another kind of disenfranchisement exists, as many poor, indigent, and immigrant children of color are denied access to programs and teaching that support their success in school. The success of the Algebra Project stands as a challenge to public school teachers, administrators, scholars, and, most important, those individuals who have traditionally advocated for the democratization of the society and schools: Will you wage a campaign for mathematical literacy, which acknowledges that every middle school student can and should learn algebra while simultaneously empowering the child's community and family? Will you organize in the spirit of Ella?

References

Comer, J. (1988). Maggie's American dream. New York: New American Library. Comer, J. (1980). School power. New York: Free Press. Delpit, L. (1986). Skills and other dilemmas of a progressive Black educator. Harvard Educational Review, 56, 379-385. Dubinsky, E. (1987). How Piaget's and related work should influence K-12 curriculum design. Unpublished manuscript. Fordham, S. (1988). Racelessness as a factor in Black students' school success: Pragmatic strategy or Pyrrhic victory? Harvard Educational Review, 58, 54-84. Henderson, A. (1987). The evidence continues to grow: Parent involvement improves student achievement. Columbia, MD: National Committee for Citizens in Education. Levin, H. (1988). Don't remediate: Accelerate. In Proceedings of the Stanford University Centennial Conference, Accelerating the Education of At-Risk Students. (Available from the Center for Educational Research, CERAS Building, 402-South, Stanford University, Stanford, CA 94305.) National Science Foundation. (1989). Materials for middle school mathematics instruction. Catalog of Federal Domestic Assistance No. 47.067, Materials Development, Research, and Informal Science Education. Peterson, J. (1989). Remediation is no remedy. Educational Leadership, 46(6), 24-25.

The Algebra Project

MOSES, KAMII, SWAP, AND HOWARD

Quine, W. V. (1981). Theories and things. Cambridge: Harvard University Press. Saxon, J. H., Jr. (1982). Algebra I: An incremental development. Norman, OK: Saxon Publishers. Schorr, L. (1988). Within our reach: Breaking the cycle of disadvantage. New York: Anchor Press/Doubleday. Usiskin, Z. (1987). Why elementary algebra can, should, and must be an eighth-grade course for average students. *Mathematics Teacher*, 80, 428-437.

to this article: Theresa Perry, Daniel Cheever, Barney Brawer, Ceasar McDowell, and finally, the teachers and administrators of the Open Program of the Martin Luther King, Jr., School.

443

This material has been reprinted with permission of the *Harvard Educational Review* for personal use only. Any other use, print or electronic, will require written permission from the *Review*. For more information, please visit www.harvardeducationalreview.org or call 1-617-495-3432.

Copyright © by the President and Fellows of Harvard College. All rights reserved.

The *Harvard Educational Review* is an imprint of the Harvard Education Publishing Group, publishers of the *Harvard Education Letter* and books under the imprint Harvard Education Press. HEPG's editorial offices are located at 8 Story Street, First Floor, Cambridge, MA 02138, tel. 617-495-3432, or email to hepg@harvard.edu.