Learning Progression for the Concept of Function: Finite-to-Finite Strand

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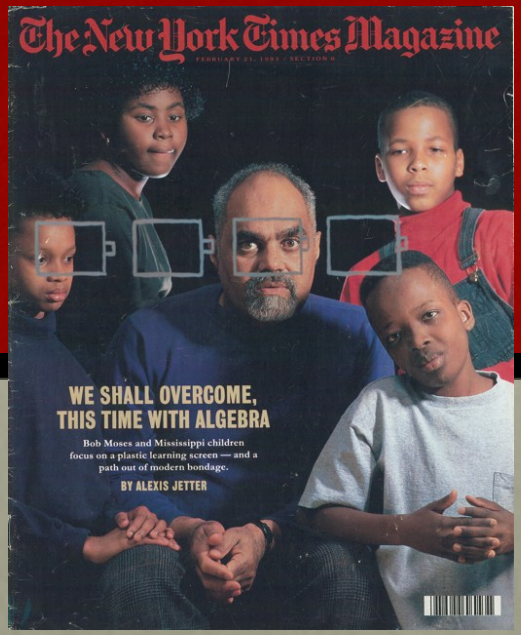
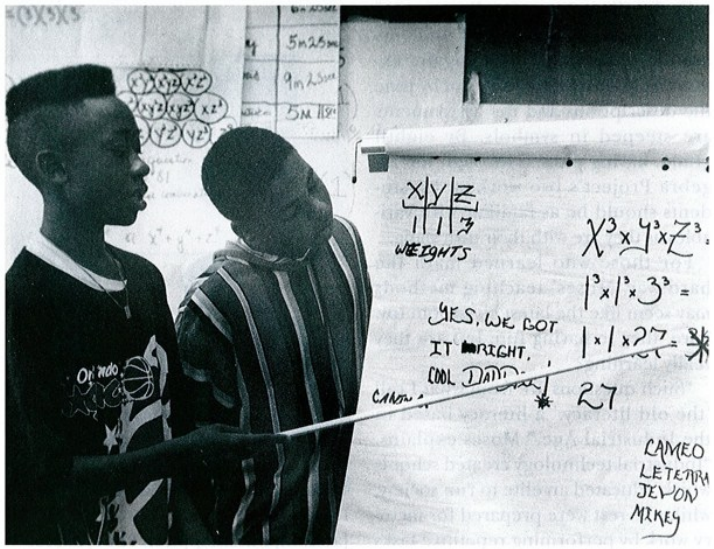
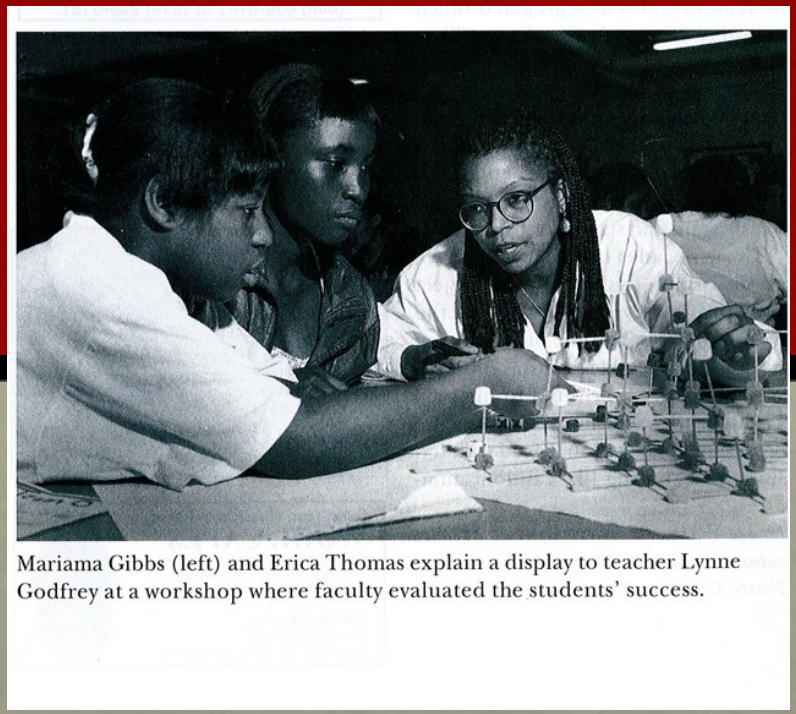
LEGACY OF THE STUDENT NONVIOLENT COORDINATING COMMITTEE (SNCC): "WE SHALL OVERCOME, THIS TIME WITH ALGEBRA"



WHAT TO TEACH. HOW TO TEACH. HOW TO ASSESS.

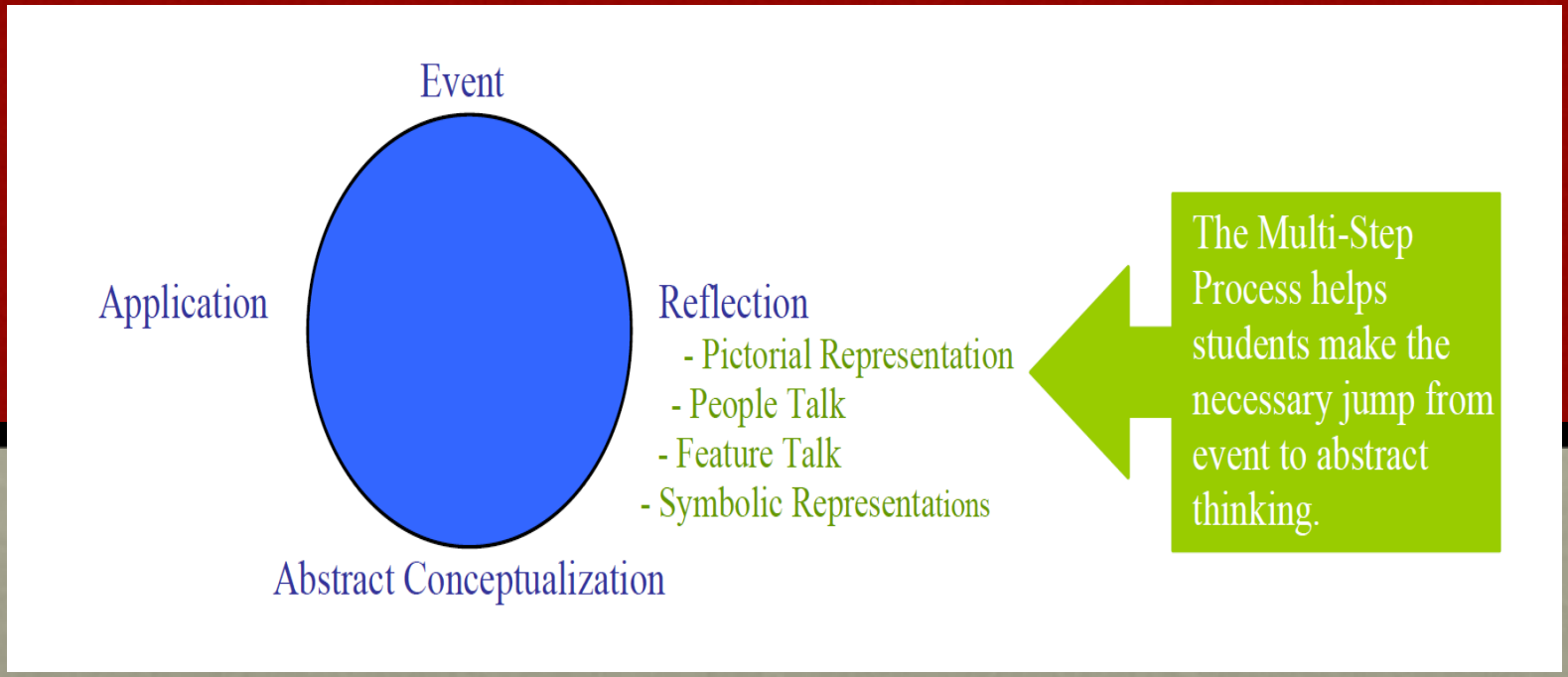


After working out a solution to a problem, seventh-graders Calvin Cain and Michael McLaurin explain to their classmates exactly how they went about it.



Mariama Gibbs (left) and Erica Thomas explain a display to teacher Lynne Godfrey at a workshop where faculty evaluated the students' success.

ALGEBRA PROJECT EXPERIENTIAL PEDAGOGY



ALGEBRA PROJECT GOAL:

- ***That students in Grades K-12 who now perform in the bottom quartile in mathematics (on standardized state exams) have the opportunity to graduate from high school able to do college or career mathematics without "remediation."***

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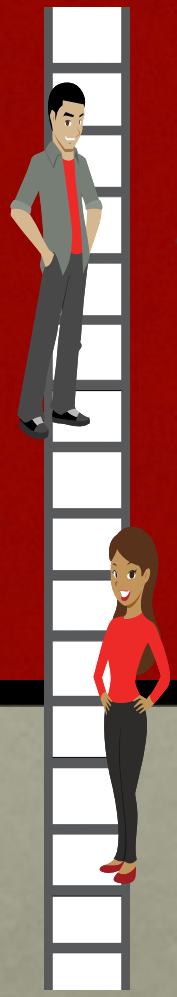
Curricular Treatment of Finite-to-Finite Functions

- Overlap between Traditional Function Strand and Finite-to-finite Strand
- Traditional treatment of finite-to-finite functions is brief, and the purpose is to demonstrate the definition of function (Akkok & Tall, 2003).
- Finite-to-finite Strand is also rooted in Algebra Project: Road Coloring Module
  - Puts students into extended contact with finite-to-finite functions
  - Distinguishing feature is that it was based on a research problem in mathematics that was unsolved at the time of its development

BUILDING LEARNING PROGRESSIONS FOR THE CONCEPT OF "FUNCTION"

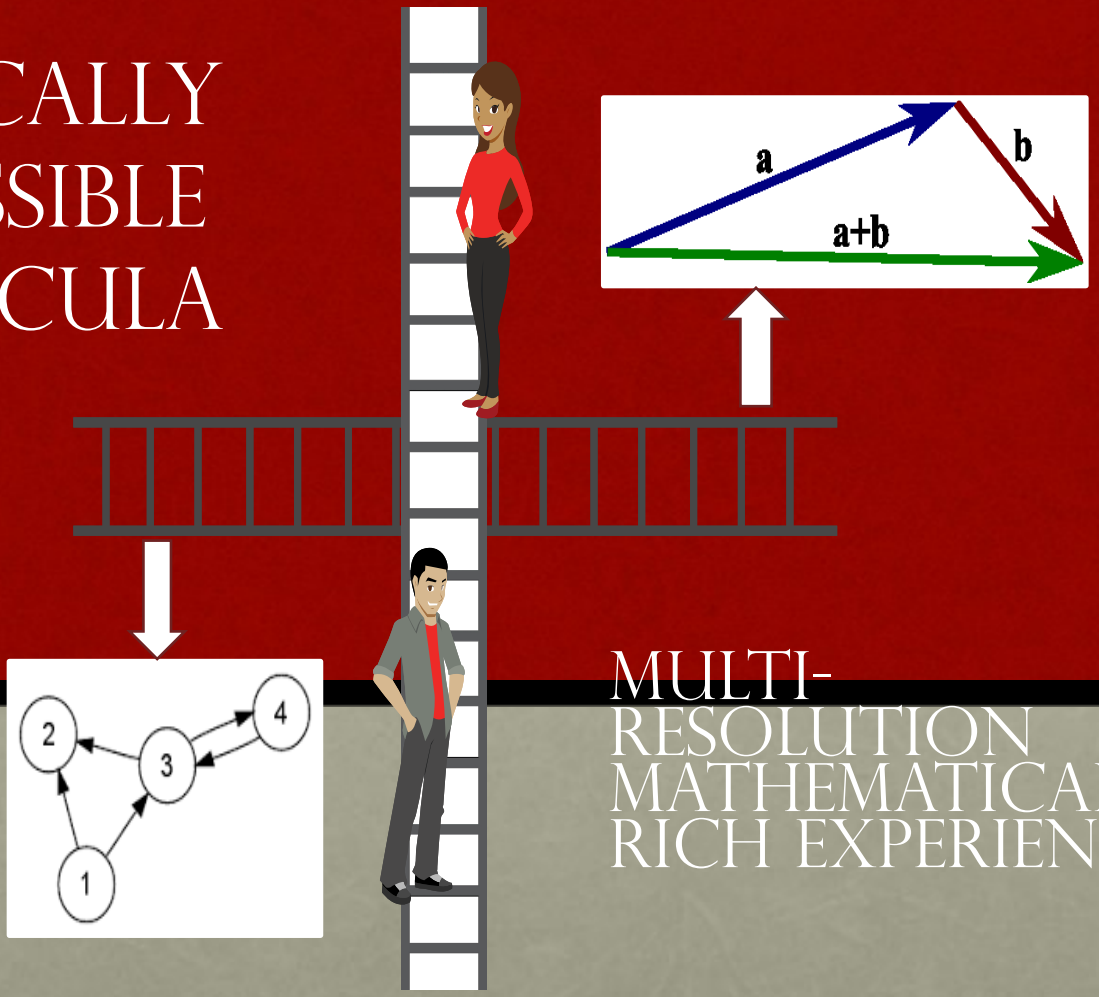


FUNCTION AS "OBJECT"

FUNCTION AS "ACTION"

CAPTURING THE FINE RESOLUTION DATA ABOUT STUDENT LEARNING

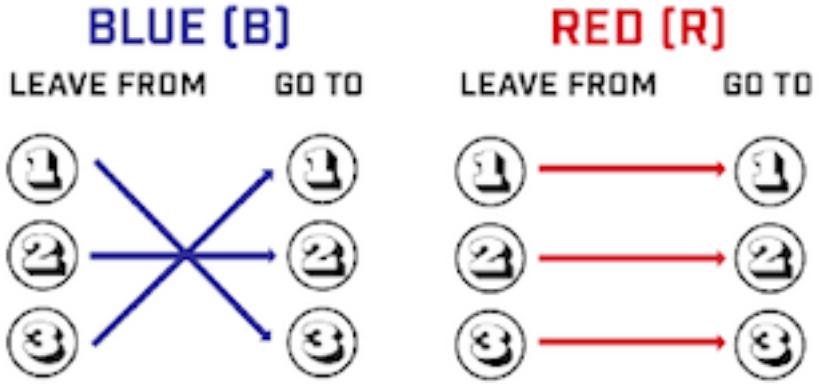
VERTICALLY ACCESSIBLE CURRICULA



MULTI-RESOLUTION MATHEMATICALLY-RICH EXPERIENCES

ROAD COLORING CURRICULUM MODULE

A city contains three buildings. There are a blue road and a red road leaving from each building in this city. These arrow diagrams represent these roads:



OVERVIEW OF LEVELS

- Level 1: Pre-function
- Level 2: Action ← A
- Level 3: Process ← P
- Level 4: "Totality"
- Level 5: Object ← O
- Level 6: Schema ← S

OVERVIEW OF LEVELS

Increasing Sophistication

- Level 1: Pre-function
- Level 2: Action
- Level 3: Process
- Level 4: Totality
- Level 5: Object
- Level 6: Schema

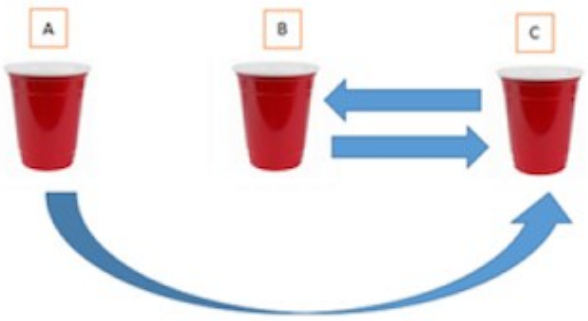
Even young children have actions, intuitions, experiences, and concepts that can be the partial foundations for the development of the function concept (Blanton, Brizuela, Gardiner, Sawrey, & Newman-Owens, 2015; Piaget, 1977).

Concepts involved with linking representations – seeing "features of function that remain the same across different representations" (Ronda, 2015, p. 305)

LEVEL 1: PRE-FUNCTION

Students have actions, intuitions, experiences, and ideas that can be partial foundations for the development of the function concept.

Each arrow in the directed graph below shows how one marble was moved from one cup to another.



1. Based on the directed graph, how many marbles are in each cup?

Cup A _0_ Cup B _0_ Cup C _3_

In a General Context

Students exhibit understanding of sequential movement; can extend a pattern.

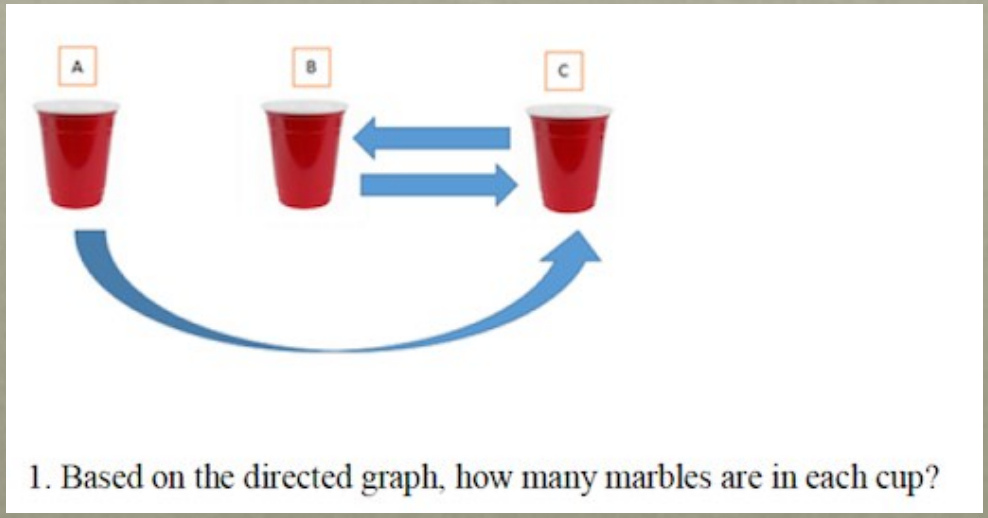
In the Road Coloring Module

A student considers only his or her own movements, without understanding that the movements within the city are simultaneous. Students may allow different color sequences for different people.

LEVEL 2: ACTION

Students see a function as an <u>action</u> requiring step-by-step instructions. Students may confound the one-valuedness condition in the definition with the uniqueness condition of the one-to-one property

Each arrow in the directed graph below shows how one marble was moved from one cup to another.



Cup A _0_ Cup B _1_ Cup C _2_

<u>In a General Context</u>

Students link:

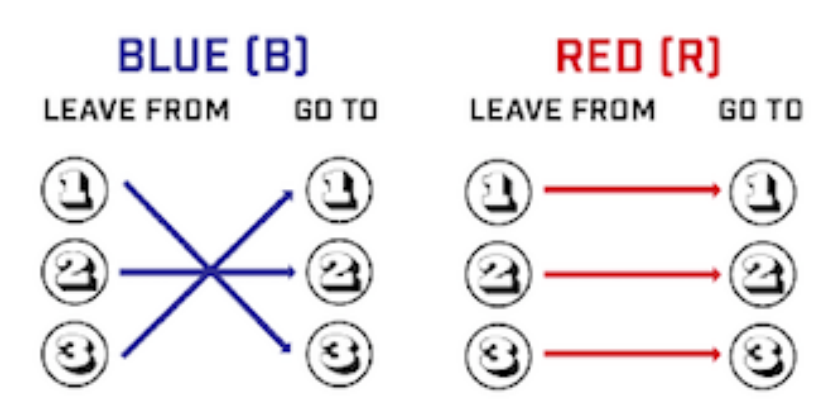
- equations with tables (Ronda, 2009)
- graphs with equations or tables by comparing individual values (Ronda, 2015)
- physical actions with arrow diagrams

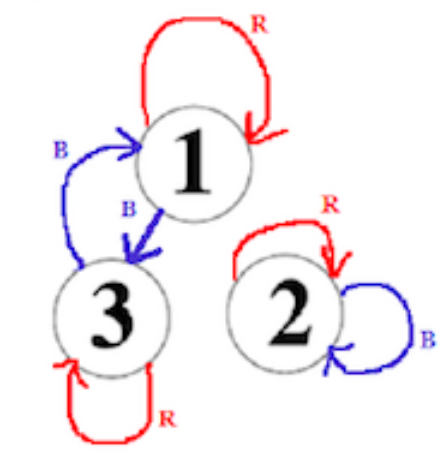
<u>In the Road Coloring Module</u>

Students track physical action of simultaneous movements, and require engagement with a representation, such as a directed graph or arrow diagram, to assist in the solution process. The function is identified with the physical movements.

LEVEL 3: PROCESS

Students make an internal mental construction called a process, which the individual can think of as performing an action, but without the need for external stimuli. Notions of one-valuedness and one-to-one property are not firmly in place (Dubinsky & Wilson, 2013).





In a General Context

Students translate between directed graphs and arrow diagrams (Dubinsky & Wilson, 2013). and link graphs with equations or tables by trends or local properties (Ronda, 2015).

May claim that the same "relationship between quantities" both is and is not a function as the representation changes.

In the Road Coloring Module

Students mentally execute instructions. The concept of dependence develops through the use of the arrow diagrams and the understanding of the relationship between the "leave from" and "go to" buildings.

LEVEL 4: TOTALITY

Students see "features of function that do not change across different representations" and "see the invariant properties of the function in a variety of representations" (Ronda, 2015, p. 305).

Notions of one-valuedness and the one-to-one property are well developed.

Concepts of domain and range are well developed.

In a General Context

Students link between various representations by invariant properties (Ronda, 2015). Students use various representations with facility in the solution.

In the Road Coloring Module

Students use multiple representations (i.e., arrow diagrams, matrices, sets of ordered pairs), and understand that these are equivalent.

LEVEL 5: OBJECT

Students see a function as an object that can be operated upon (see Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Cottrill, Dubinsky, Nichols, Schwingendorf, Thomas, & Vidakovic, 1996)

Concepts of domain and range are extended to composition of functions.

In a General Context

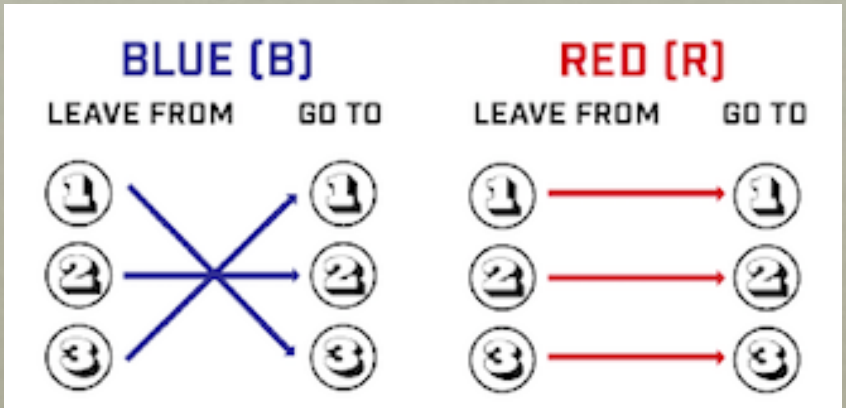
Students engage in actions on representations that result in the construction of new functions (Ronda, 2015).

In the Road Coloring Module

Students use symbolic representations to operate with functions.

Students compose functions; students multiply matrices and understand how the multiplication relates to the composition of functions.

LEVEL 5: OBJECT



[B] * [R] =

Leave From \ Go to	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

Explain how you determined your answer.

I combined the Red and blue Arrow diagram together and followed the arrows from the Leave from to Go to

b. What is the meaning of [B] * [R] in the context of the city?

B * R is when there trying to Say where does the person land if you combined the blue Arrow with the Red arrows